2023

NC(SMC)²
Accuracy Round

Accuracy Round

Instructions: On this round, you will have 40 minutes to answer 10 questions. Each correct answer will contribute 2 points to your total score and questions that are incorrect or left blank will be marked as 0 points. Submit your answers here: https://tinyurl.com/ncsmc2accuracy. Good luck!

1. If $x = 20$ and $y = 23$, what is the value of the expression $5x - 13y$?

2. Assume that 1 meter = 1.1 yards. If a swimmer can swim 100 meters in 110 seconds, at the same rate, how fast can she swim 100 yards?

3. A rectangle has a perimeter of 40 and an area of 96. What is the square of the length of the diagonal of this rectangle?

4. Sheep B runs a grass mowing (eating?) business. For each customer, he is paid a base price of \$7. On top of that, he receives \$11 for every hour he spends eating. Assume that Sheep B works a positive integer number of hours at each customer. If Sheep B made \$94 today, how many hours did he work?
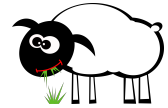
5. Ana's birthday is 1/1/2000, Bob's birthday is 1/1/2002, and Carla's birthday is 1/1/2009. During what year did Ana, Bob, and Carla's ages form a Pythagorean triple (that is, the numbers could be the side lengths of a right triangle)?

6. How many positive integer factors does $3^8 - 1$ have?

7. If 6 fair coins are flipped, the probability that there are more heads than tails can be expressed as $\frac{m}{n}$ for relatively prime positive integers m, n. What is $m + n$?

8. Albert has 2 pairs of socks in a drawer: 1 black pair and 1 white pair. Every second, he removes a sock randomly from the drawer, until he has matching socks. The average number of seconds Albert must wait until he has a matching pair of socks can be expressed as $\frac{m}{n}$ for relatively prime positive integers m, n. What is $m + n$?

9. Let ABC be a triangle with side lengths $AB = 13$, $BC = 14$, and $CA = 15$. Let G be the centroid (the intersection of the three medians, lines that connect vertex to midpoint of sides) and let H be the orthocenter (the intersection of the three altitudes of the triangle). The length of GH can be expressed as $\frac{\sqrt{m}}{n}$ for some squarefree positive integer m that is relatively prime to n. Find $m + n$.
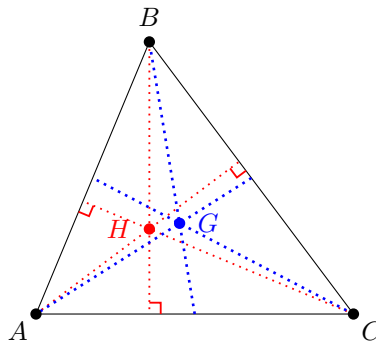
10. Let a, b, c, d be pairwise distinct positive integers such that $a, b, c, d \le 100$. Find the maximum possible value of $\gcd(a, b) + \gcd(b, c) + \gcd(c, d) + \gcd(d, a)$.